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Design & Construction Of an Inertia Governor

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DESIGN AND CONSTRUCTION OF AN INERTIA GOVERNOR

BY

CHARLES HENRY GREEN

THESIS FOR THE DEGREE OF BACHELOR OF SCIENCE
IN MECHANICAL ENGINEERING

IN THE
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CHARLES HENRY GREEN

ENTITLED DESIGN AND CONSTRUCTION OF AN INERTIA GOVERNOR.

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE

OF Bachelor of Science in Mechanical Engineering.

L. P. Breckenridge

HEAD OF DEPARTMENT OF Mechanical Engineering.

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Design and Construction of an Inertia Governor for the Nagle Engine located in the U. of I. Steam Laboratory.

I. Introduction.

a. Statement of Problem.

The problem proposed in this thesis is the design of a governor for the Nagle engine located in the U. of I. steam laboratory. The engine is a 6"×7" vertical, non-condensing, single engine. As it stands, it has a throttle governor, and it is proposed to design and construct a governor of the inertia type to replace this throttle governor. The speed fluctuation during a revolution is not to be more than 1% and the revolutions per minute are 250. The engine is to run between zero and 16 H.P.

b. Historical Sketch.

Previous to the analysis of the type of governor to be employed in this construction, it may be well to note the development of the inertia governor.

Until the year 1845, the only method of regulating the speed of engines, (with the exception of the Corliss gear) was by throttling the steam pressure by means of common fly-ball governors. There were in use at this time some modified forms of the modern centrifugal governor which acted upon the valve mechanism to regulate the speed; but these were found to be but little better than those of the throttle governor type. In 1845, Werner and William Siemens suggested a construction for a governor, by which the acceleration or retardation of a rotating mass due to a change in load might be used to move the valve mechanism of an engine, and in the opinion of the inventor only an infinitesimal change of velocity would be required for a change from no load to full load.

The arrangement proposed by Siemens is shown in Fig. 1. The shaft E is connected in some way to the main shaft of the engine, so that its speed is proportional to that of the engine itself

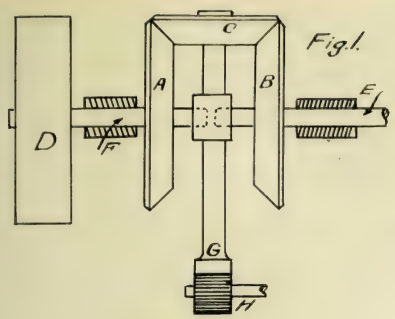


Fig. 1.

The shaft F is driven by E through the bevel gears A, B, and C. C is a gear free to turn about its own axis or the axis EF. The hub that carries the axis of the gear C also carries a spur segment G which is in contact with the spur wheel H. On the shaft F is keyed a heavy mass D. If G is held stationary, F will revolve with the same speed as E. If the speed of E is lessened F will continue to move with its original angular velocity. This causes the gear C to move about the axis EF and by virtue of this rotation about EF causes the spur wheel H to revolve. H is in turn connected in such a way to the valve mechanism that any change in its position causes the length of valve travel and cut-off to be changed. The disadvantage with this arrangement is that the engine governs at the speed of the shaft F, and if for any reason, the speed of F is

reduced, the speed of the engine is reduced likewise. Then in order to run at a given speed, the engine must be brought up to the desired speed by external means before the governor can control it at that speed, which of course is impracticable. Briefly stated the Siemens' principle is this: The acceleration of a free rotating mass due to a change in load or rather the resistance to this acceleration, (the inertia resistance of the mass), may be employed as a force to operate or shift the valve mechanism of an engine. Regulation based upon this principle is known as inertia regulation.

Other methods of applying this principle were soon adopted. A disc, rotating freely about the engine shaft, which connected directly to the valve gear was employed. Following this, various linked mechanisms were substituted, in such a way that the lagging mass would give the eccentric a smaller throw and vica-versa. These arrangements still had the same fault

as the Siemens differential gear arrangement, in that they would not keep the engine at a fixed mean speed. Later a governor was devised by F. M. Rites, which, besides being an inertia governor, had a weak centrifugal effort capable of bringing the engine up to the mean speed and keeping it there, but depending for its regulation upon the inertia action. The Rites principle is used almost universally where inertia regulation is employed, and as an inertia governor is to be designed for this thesis one of this type will be considered.

II Analysis of the Inertia Governor

a. The Inertia Forces.

Preliminary to the design an investigation of the various forces is necessary. Take the general case of a mass pivoted at a point of a second mass which rotates about a fixed axis. Thus in Fig. 2, let S be the center of rotation of the mass a, while the mass b rotates about O, a point in a. Let G be the center of gravity of the mass b, and P any point in it.

let ω = the angular velocity of a about S.

" Ω = " " " " " " " b " " O.

" $\alpha = \frac{d\omega}{dt}$ " " " acceleration of a.

" $u = r\Omega$ = linear velocity of P relative to O.

" J = the polar moment of inertia of b. about G.

Let $M = \frac{W}{g}$ = mass of b.

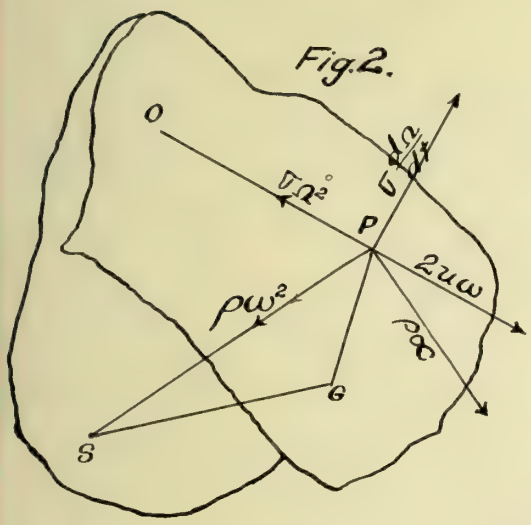


Fig. 2.

The acceleration of the point P, according to the law of Coriolis for the acceleration of relative motion, has three components, viz: (1) the acceleration of P rotating about O with the mass a stationary, (2) the acceleration of the point of a coincident with P; and (3) a component along OP (see Fig. 2). The first of these accelerations may be resolved into components acting radially along OP equal to $r\Omega^2$ where $r = \overline{OP}$ and a component $r\frac{d\Omega}{dt}$ acting perpendicular to the line OP at P. The second of these accelerations can be resolved into a radial component along PS equal to $\rho\omega^2$, where $\rho = \overline{SP}$ and a

and a tangential component $\rho \frac{dw}{dt} = \rho\alpha$ perpendicular to PS. The third acceleration $2vw$ acts along OP.

Let ΔM denote an element of mass at the point P. If the acceleration components are each multiplied by ΔM the products are the effective forces acting on the element at P and by D'Alembert's principle these forces reversed (the so-called inertia forces are in equilibrium with the external forces acting on the particle. If now we sum up these inertia forces for the entire mass, the resultant system of inertia forces will be in equilibrium with the system of external forces acting on the mass. For convenience in the summation process, the forces at P, Fig. 3. can be resolved into components perpendicular and parallel to to SG and PG respectively. Thus in Fig. 3,

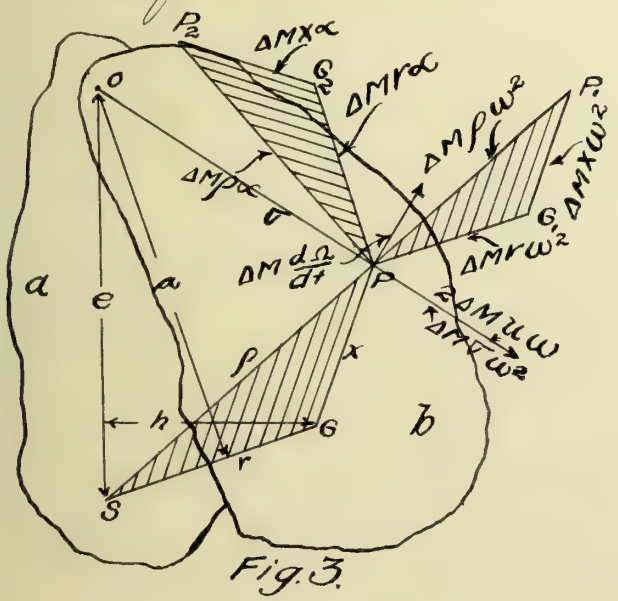


Fig. 3.

The △ PSG and PP'G' are similar as their sides are parallel. The △ PSG and P_2PG_2 are similar as their sides are perpendicular to one another.

Then $\rho : r :: \Delta M\rho\alpha : G_2P$

$$G_2P = \Delta M\rho\alpha\frac{r}{\rho} = \Delta Mr\alpha$$

In like manner $P_2G_2 = \Delta Mx\alpha$

Since P_2G_2 and G_2P are forces acting at P, on the components into which $\Delta M\rho\alpha$ is resolved. By the similarity of △ SPG and PP'G', and since $PP' = \Delta M\rho\omega^2$

$$\overline{PG_1} = \Delta Mr\omega^2$$

$$\overline{P_1G_1} = \Delta Mx\omega^2$$

are the components into which $\Delta M\rho\omega^2$ is resolved. The components $\Delta Mv\omega^2$ and $2\Delta Mz\omega$ need not be taken into consideration as they act along OP and do not affect the motion except to increase the friction at the pivot O. The force $\Delta Mv\frac{d\Omega}{dt}$, being insignificant compared with the other forces, need not enter into the calculations.

Take the system of forces $\Delta Mx\omega^2$. By resolution into rectangular components, the forces $\Delta Mx\omega^2$ give a resultant and a resulting moment each equal to zero.

The second group of forces $\Delta M r\omega^2$ are parallel to the radius SG and are proportional to the masses ΔM. Their resultant is the centrifugal force, and passes through the center of gravity, and is directed outward from the center of the shaft S. The resultant then is

$$r\omega^2 \Sigma \Delta M = M r \omega^2$$

and its moment about the pin O is

$$M = M r a \omega^2 = 2(\text{area } \Delta SOG) M\omega^2 = M e h \omega^2$$

where a is the moment arm from O to SG.

Consider the forces $\Delta M x \alpha$. When resolved into rectangular components, the resultant equals zero for each set of components, but the system does not reduce to zero but to a couple, the moment of which about G is.

$$M_i = \alpha \Sigma x^2 \Delta M = J\alpha$$

where J is the polar moment of inertia of the mass about G. The components $\Delta M r \alpha$ are summed up into a resulting force $M r \alpha$ acting at G, with a moment about O

$$M_i' = M r c \alpha$$

where c is the moment arm from O to the force $M r \alpha$. Hence the total amount of the inertia forces is

$M_j = J\alpha + Mrc\alpha = J'\alpha$ where $J' = J + Mrc$.

The forces of the governor being now determined, it will be necessary to determine the relation between them for the important functions of the governor and engine.

b. Stability of Regulation.

The following discussion is essentially a translation of a continued article in the Zeitschrift des Vereins Deutscher Ingenieure, May 6-20, 1899.

The equilibrium between load and effort on the part of the engine is necessarily disturbed, and there will be alternate stages of acceleration and retardation. To these variations between load and effort the governor must adjust the flow of steam to keep the speed constant. A movement of the governor from an equilibrium position will be succeeded by a series of oscillations of the governor mass. The better the governing action, the fewer are the oscillations of the governor and the smaller their magnitude. The arrangement of an inertia governor may be such that the oscillations never

cease and sometimes increase in magnitude, and after a disturbance, the governor may never arrive at a state of rest relative to the engine. In a case of this kind the regulation previous to the disturbance was unstable. Stability is self-evidently, the first condition to be placed upon the governing device.

In order to simplify the following discussion the center of gravity of the governor masses will be taken as a characteristic point and all masses reduced to that point. The same method will be used in connection with the engine. These points have but one degree of freedom and follow the law of rectilinear motion, i.e. mass × acceleration = moving force. In regard to the governor forces, it is only necessary to consider the relative motion; and the forces acting are:-

1. Spring Tension
2. Centrifugal Force
3. Total Inertia Force.
4. Friction.

The point of concentration of the masses

is not necessary as all that is desired is the effort and resistance.

In Fig. 4 the effective governing forces and the effective efforts of the motor are

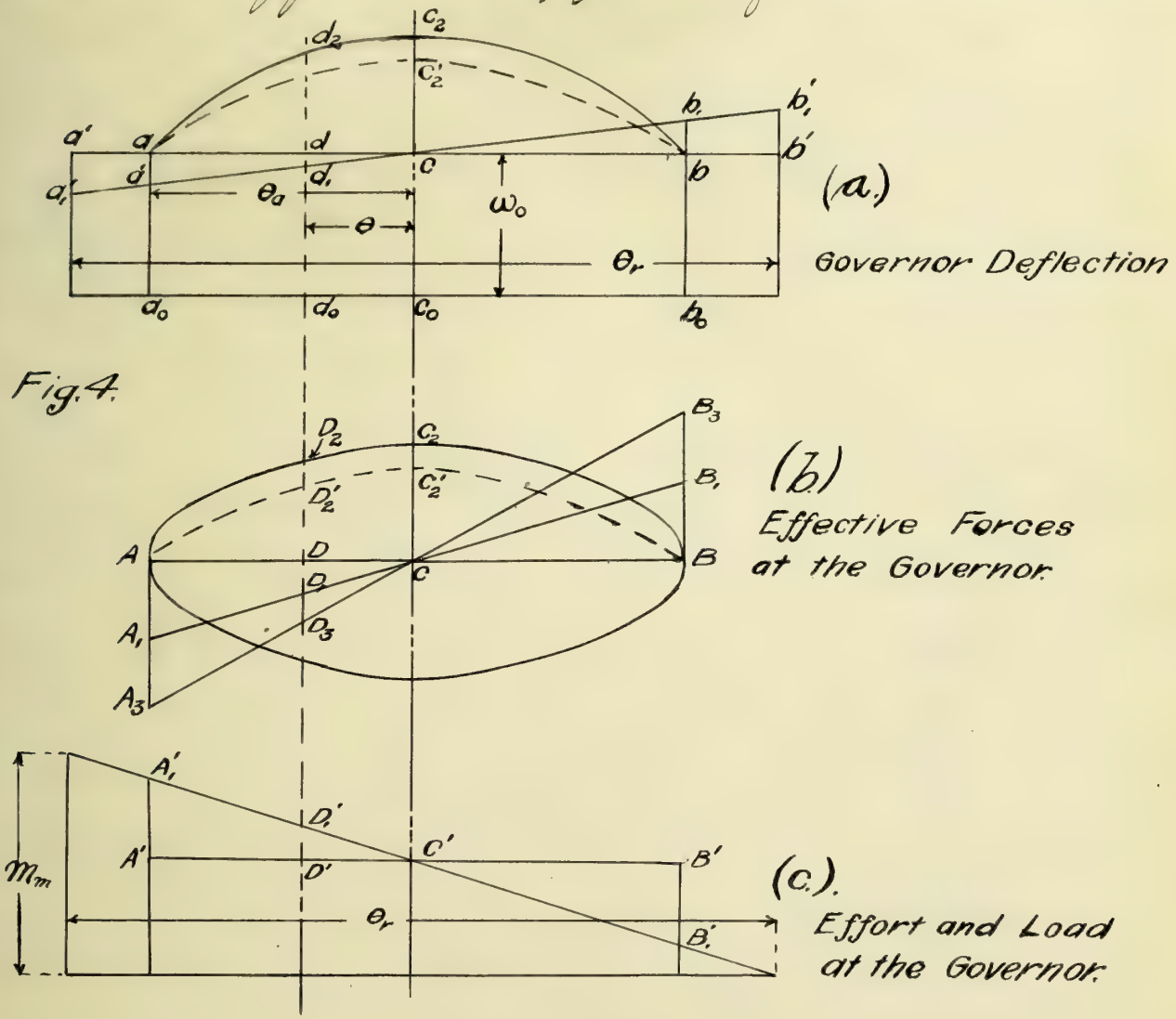


Fig. 4.

laid off from an axis ACB ($\bar{o}$) which represents the governor travel. The direction AB represents an increasing deflection and in (a), $\omega^\circ = CC_0$, the equilibrium angular velocity for the position C of the

governor. The regulation is static, the forces being in equilibrium, that is, the spring tension is equal to the centrifugal force and a, b, represents the velocity curve for the point of the governor travel under consideration. The intercepts a'a, and b'b, determine in a known manner the coefficient of fluctuation in the speed of the motor Now during its transition from A to B, the governor occupies the position O and the velocity corresponding to O = $d_0 d_2$. The velocity $d_0 d$, corresponds to equilibrium since the the curve a, cb, is the velocity curve for equilibrium for this transition period, and there remains d, d_2 which is in excess and gives rise to a governing force directly proportional to $d, d_2 = O, O_2$ (3). The force O, O_2 is composed of two forces O, O and $O O_2$ of which O, O corresponds to the excess of velocity $d d$, which would be necessitated if the governor were moved from the position C (with velocity of w_0) to the position O. This is a static governing force which is held in equilibrium by the spring. The second part $O O_2$ is proportional to the velocity excess $d d_2$ and is termed the dynamic governing force.

The effort curve to be considered is shown in (c) and is represented by $A'C'B'$, which is assumed to be a straight line. In (c) for the position D there is an excess of the effort represented by $D'D$. This increase of effort does not mean the periodic variation of effort and load of the steam engine however. The excess D, D' produces a certain angular acceleration and therefore a tangential inertia force at the governor and a moment $M_s = J\alpha + Mr_c\alpha$ which, when reduced to the center of gravity, is represented by O_1O_3 (b). If the deflection O_1O' is small, O_1O_3 may be taken as proportional to the force ΔP, the excess of the effort over the load, and may be added directly to the static governing force. Thus the tangential inertia force consists in a proportional increase of the static governing force.

The limit of stability is the state in which the governor oscillates isochronously about its point of equilibrium, and in the same way the velocity of the motor oscillates about its value corresponding to equilibrium.

If an appreciable time lapses between the disturbance of equilibrium of the governor and velocity of the motor, the oscillations will be

harmonic, that is, they will vary according to the sine law as functions of the time. The speed of the motor and the space described by the governor are sine functions with equal periods; then the velocity as a function of the space may be represented by an ellipse. Let AC = CB be the amplitude of the governor point which it has reached after having started from some original position. Passing from A to C there is an excess of effort of the motor and therefore the velocity must increase and reach its maximum at C as shown by Fig. 4(a). The ellipse representing the velocity is aC_2b (a). The governor speed according to the theory can be represented by an ellipse $aC_2'b$, a and b being taken as the points which correspond to zero velocity during the period of transition.

From what has been stated D_3D_2 (b) is the resultant of the spring tension and the inertia forces on the governor. To these forces is yet to be added the resistance of the valve gear, which for the present purpose may be assumed to be proportional to the ordinates of the ellipse $aC_2'b$ and may be represented on (b) as $AD_2'C_2'B$. Since the motion of the governor point

is harmonic, the resultant force is proportional to the distance of the governor point from C. This will occur when the arrangement of the resistance is such that C_2' coincides with C_2, and for the limit of stability, the dynamic force is just neutralized by the brake resistance.

To obtain an analytic expression for the condition of stability we will pass from forces to moments and from masses to mass moments.

Let J_s denote the moment of inertia of the oscillating masses of the motor.

" M_m, the greatest moment of the effort

" ΔM_a, the excess of the moment of the effort over the moment of the load in the position A.

" θ_a, the angular deflection of the governor weight from position C toward A.

" θ_2, the total deflection of the governor weight.

" δ, coefficient of fluctuations in the speed.

" $\delta_1 = \frac{\theta_a}{\theta_2}\delta$ = change in velocity between C and A expressed in percent of ω_0.

" M_0, the moment of the spring tension about the center of rotation of the governor weight for the position C.

If the governor be considered at rest the effective moment M_e for the position 3 is the product of the static moment M_0 and double the excess of the velocity $\frac{d_1 d_2}{d_0 d_1}$, as shown by the Theorem of Lasteus and Lang.

$$M = b\, d_0 \overline{d_2}^2 \quad , \quad M_0 = b\, d_0 \overline{d_1}^2$$

$$M_e = M - M_0 = b\left(d_0 \overline{d_2}^2 - d_0 \overline{d_1}^2\right)$$

$$= b\,(d_0 d_1 + d_0 d_2)\, d_1 d_2 = 2b(d_0 \bar{d}_1 \cdot d_1 \bar{d}_2) \text{ approx.}$$

$$\text{Thus } \frac{M_e}{M_0} = \frac{2b \cdot d_0 \bar{d}_1 \cdot d_1 \bar{d}_2}{b\, d_0 \overline{d_1}^2} = 2\frac{d_1 d_2}{d_0 d_1}$$

or approx. from Fig. 4. $\frac{M_e}{M_0} = 2\frac{d_1 d_2}{c_0 c} M_0$ (1.)

From Fig. 4 $d_1 d_2 = d_1 d + d d_2$; $d d_1 = \frac{\theta}{\theta_r}\delta$; $d d_2 = \omega - \omega_0$ (2)

and with the moments of the static and dynamic forces

$$D D_1 = 2\delta \frac{\theta}{\theta_r} M_0 \qquad (3).$$

and $D_1 D_3 = J'\left(\frac{d\omega}{dt}\right)_0$ where J' is the (4)

expression derived on page . Now for the motor masses we have.

$$J_s \frac{d\omega}{dt} = \Delta M = \frac{\theta}{\theta_r} M_m \text{ from Fig. 4 c.} \qquad (5).$$

$$\frac{d\omega}{dt} = \frac{\theta}{\theta_r} \frac{M_m}{J_s} \qquad (6).$$

From this follows the angular acceleration for the position 3 and finally the effective moments of the governor. From (3), (4), & (6) we have

$$D D_3 = 2\delta \frac{\theta}{\theta_r} M_0 + \frac{J'}{J_s} \frac{M_m}{\theta_r} \theta = a\theta \qquad (7).$$

where a denotes the factor,

$$a = 2\frac{\delta}{\theta_r} M_0 + \frac{J'}{J_s}\frac{M_m}{\theta_r} \qquad (8)$$

Since the valve resistance neutralizes the dynamic force for the limit of stability,

$$J'\frac{d^2\theta}{dt^2} = -a\theta \qquad (9)$$

From (9) $\quad \frac{d^2\theta}{dt^2} = -\frac{a}{J'}\theta \qquad$ Let $\frac{a}{J'} = K^2$

Then $\quad \frac{d^2\theta}{dt^2} = -K^2\theta$

$$\theta = C_1 \cos Kt + C_2 \sin Kt \qquad (a)$$

Let $\theta = \theta_a$ when $t = 0$, then $\theta = \theta_a \cos Kt$

Substitute in (a), then $0 = C_1(0) + C_2 \sin 90°$

$C_2 = 0$ and $C_1 = \theta_a$

$\therefore \theta = \theta_a \cos Kt$. which is the (10) equation of harmonic motion or oscillation.

$$\frac{d\theta}{dt} = K\theta_a \sin Kt \qquad K_2 = \frac{a}{J'} \qquad (11)$$

Then as the variables ΔM and ΔM_a are also functions of the time with equal periods as the governor oscillates

$$\Delta M = \Delta M_a \cos Kt$$

Then from eq. (5) by integration

$$J_s(\omega - \omega_0) = \Delta\frac{M_a}{K}\sin Kt$$

The velocity increment at c is $\Delta\omega$ and

$$\sin Kt = \sin 90° = 1$$

Then $\Delta\omega = \frac{\Delta M_a}{J_s K}$, $\Delta M_a \frac{\theta_r}{\theta_a} = M_m$, $\Delta M_a = \frac{\theta_a}{\theta_r} M_m$

$$\Delta\omega = \frac{\Delta M_a}{J_s K} = \frac{\theta_a}{\theta_r}\frac{M_m}{J_s K} \qquad (12)$$

The effective moment at C is then

$$M_{e(c)} = 2\frac{\Delta W}{\omega_0} M_0 = 2\frac{\theta a\, M_m M_0}{\theta r\, J_s K \omega_0} \quad \text{from (12).}$$

If the resistance takes the form of a dash-pot with moment about c $M_{b(c)} = K\left(\frac{d\theta}{dt}\right)_c = K K \theta_a$, its moment is balanced by $M_{e(c)}$.

$$M_{b(c)} \geqq M_{e(c)}$$

Substituting, $K^2 \geqq \frac{2 M_m M_0}{J_s K \omega_0 \theta r}$ (13).

From (11) and (8),

$$K^2 = \frac{a}{J'} = \frac{1}{J'}\left(\frac{2\delta}{\theta r} M_0 + \frac{J'}{J_s}\frac{M_m}{\theta r}\right) \quad (14)$$

Then eq. (14) $\geqq$ Eq. (13) and

$$\frac{2 M_m M_0}{J_s K \omega_0 \theta r} \geqq \frac{1}{J'}\left(\frac{2\delta}{\theta r} M_0 + \frac{J_1}{J_s}\frac{M_m}{\theta r}\right) \text{ and}$$

$$\frac{K\theta r}{2 M_0}\left[\frac{\delta J_s \omega_0}{M_m} + \frac{J'\omega_0}{2 M_0}\right] \geqq \frac{J'\theta r}{2 M_0} \quad (15).$$

Simple physical meanings are given to these terms.

$T_0 = \frac{J_s}{M_m}\omega_0$, since $M_m = J_s \cdot \frac{\omega_0}{T_0}$ where J_s is the moment of inertia of the oscillating masses of the motor and M_m is the maximum moment of the effort of the motor. T_0 then represents the time taken for the moment M_m to act to bring the motor masses from zero velocity to ω_0.

$T' = \frac{K\theta r}{2 M_0}$, $M_0 = K\frac{d\theta}{dt}$, $\frac{d\theta}{dt} = \frac{M_0}{K}$ = angular velocity.

Since $\theta r = \frac{M_0}{K} \cdot 2T'$, $2T'$ is the time required to make the stroke θr with the angular velocity $\frac{M_0}{K}$.

$T^2 = \frac{J\theta_r}{2M_0}$, $\frac{d^2\theta}{dt^2} = \text{Moment} = \frac{M_0}{J}$, then $\frac{d\theta}{dt} = \frac{M_0}{J}.T$

$\int_0^{\theta_r} d\theta = \frac{M_0}{J}\int_0^T T\,dt$, $\theta_r = \frac{M_0}{J}.\frac{T^2}{2}$

Then $\theta_r = \frac{M_0}{J}.\frac{(2T)^2}{2}$, hence $T^2 = \frac{J\theta_r}{2M_0}$ showing that $2T$ is the time of fall of the governor or the time required to sweep over the angle θ_r

$T'' = \frac{J'\omega_0}{2M_0}$, $\omega_0 = \frac{M_0}{J'}.2T''$, $\frac{d\omega}{dt} = \text{Moment} = \frac{M_0}{J'}$, $\omega_0 = \frac{M_0}{J'}T''$

Here $2T''$ denotes the time during which M_0 accelerates the equivilent of the governor masses as regards the inertia action from zero to ω_0.

Equation (16) can now be more simply written using the symbols just obtained.

$$T'(\delta T_0 + T'') \geqq T^2. \qquad (16a).$$

The previous equations were obtained by considering the resistance to follow the law governing the motion as is the case of the ordinary dash-pot. In the case where the friction of the valve gear, forms the damping action to annull or partially destroy, the dynamic force, the friction is assumed as constant throughout the deflection. Fig. (5) is given the same notation as Fig. 4. as regards the governing forces and efforts. In Fig. 5. (a) the inertia force is O_3O_1, the static force O_1O, the dynamic force OO_2, and there is a constant frictional resistance O_3O_4. The free forces

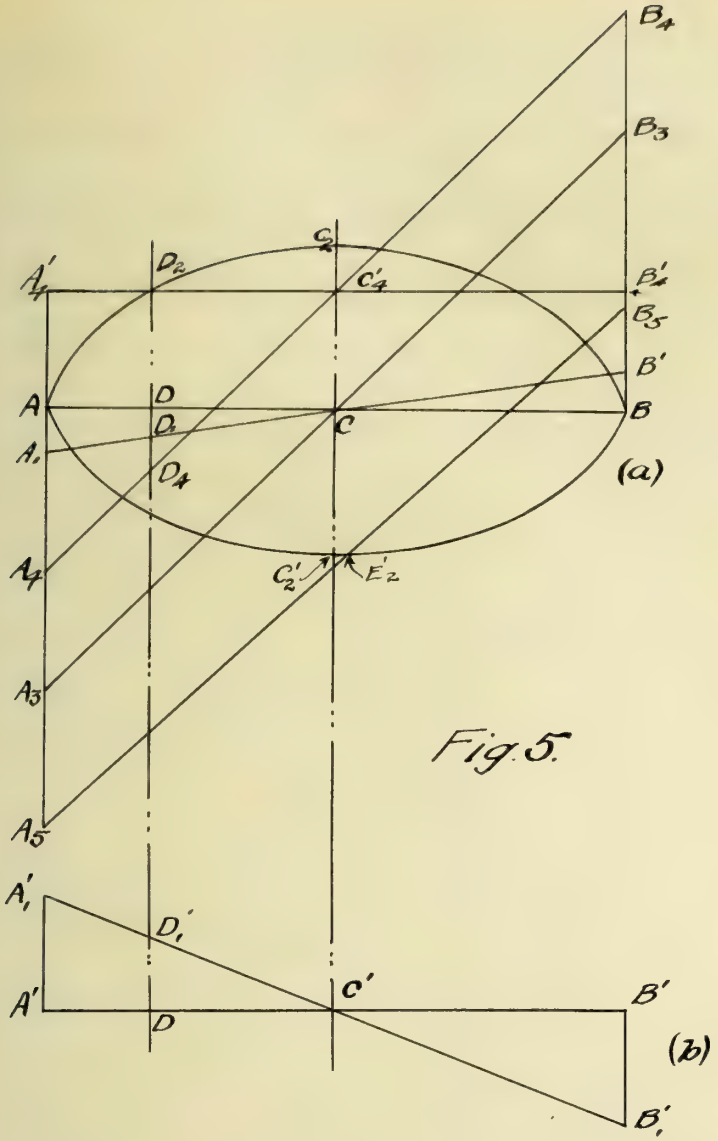


Fig. 5.

are represented by the ordinates of the areas $A_4 A D_2 E_2 D_4$ and $E_2 B_4 B$, the first corresponding to positive and the second to negative acceleration. The limit for stable regulation in this case may be considered that state in which the governor point vibrates between the points A and B and all other quantities experience similar changes.

The conditions for such periodic motion may be derived as follows: Assume $\bar{AC} = \bar{CB}$. The areas $\triangle A_3 C$ and $C B_3 B$ are the sum of the dynamic and static forces. There remains the positive work $\triangle C_2 B$ of the dynamic force, and the negative work

$A_4 B_4 B_3 A_3 = A A_4' B_4' B$ of the frictional force which is assumed constant. The governor starts from relative rest at A, and comes to rest at B and the area $A A_4' B_4' B$ = area AC_2B. As before AC_2BC_2 was assumed as an ellipse or the curve is assumed to have a horizontal tangent at C_2. The area of the half ellipse is $\frac{\pi}{2} \cdot \overline{AC} \cdot \overline{CC_2}$, then see (Fig. 5)

$$\frac{\pi}{2} \cdot \overline{AC} \cdot \overline{CC_2} \geq \overline{AB} \cdot \overline{CC_4}$$

$$\text{or } \overline{CC_4} \geq \frac{\pi}{4} \overline{CC_2}$$

In the previous discussion

$$\overline{CC_2} = 2 \frac{\Delta w'}{\omega_0} m_0 = m_e, \text{ the effective effort}$$

CC_4 is the force necessary to overcome friction, and denoting by $\Delta w'$ the resulting change in velocity,

$$CC_4 = 2 \frac{\Delta w'}{\omega_0} m_0 \qquad (16)$$

In the limit $\Delta w'$ is the insensibility and if we the coefficient of insensibility as the quotient,

$$\epsilon = 2 \frac{\Delta w'}{\omega_0}, \quad CC_4 = \epsilon m_0$$

From (13) by the condition of stability,

$$\overline{CC_4} \geq \frac{\pi}{4} \overline{CC_2}$$

$$\epsilon m_0 \geq \frac{\pi}{4} \left(2 \frac{\Delta w'}{\omega_0} m_0\right)$$

$$\therefore \epsilon \geq \frac{\pi}{2} \frac{\Delta w}{\omega_0} \qquad (16a)$$

Then from (12) and (13),

$$\epsilon \geq \frac{\pi}{2} \cdot \frac{m_m}{J_s \omega_0 K} \cdot \frac{\theta_a}{\theta_r} \qquad (17)$$

which in the notation of (15a) becomes by squaring

$$\epsilon^2 \geq \frac{\pi^2}{4} \cdot \left(\frac{\theta_a}{\theta_r}\right)^2 \cdot \frac{T^2}{T_0(\delta T_0 + T'')} \qquad (17a)$$

In this case, the inertia action T" renders possible the employment of an isochronous governor ($\delta = 0$), and it is possible to even make δ negative. In the latter case the speed increases with the load. The limit to the case with friction as the damping means is given by the minimum inclination of the force line $A_3 B_3$. Let us assume for a given case the proportions so chosen (see Fig. 6), that A_4 falls at A which signifies that the smallest necessary coefficient of insensibility is equal to the coefficient of fluctuation for the part of the governor's movement under consideration. Stability is still preserved as the friction area $A B_4 B_3 A_3$ is equal to the work area $A C_2 B$. Let the insensibility, and with it the friction be increased to $A A_5$. The motor speed must first

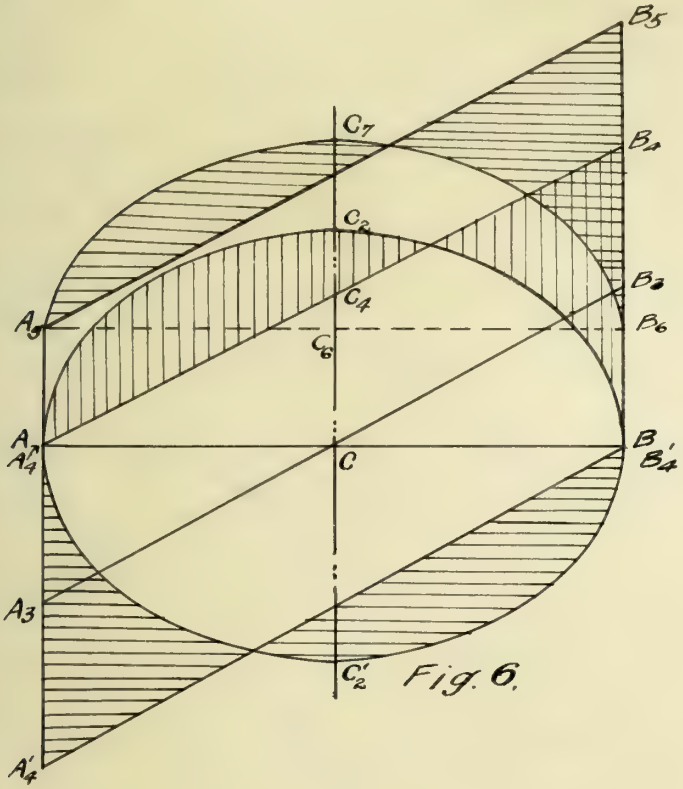


Fig. 6.

increase until the dynamic force has reached the value AA_5, then the governor first begins to move. The succeeding motion will be the same as that in the first case. The motor speed increases according to the same law and the dynamic force is represented by the curve $A_5C_7B_6$ congruent to AC_2B. The work of friction $A_3A_5B_5B_3$ and that of the force $AC_5C_7B_6B$ are equal and opposed, and at B, the governor comes to rest. A pause then ensues until the velocity w_0 decreases by an amount AA_5 when the operation is again repeated. To decrease the fluctuation, (by increasing the instability), the insensibility being greater than is necessary, we may rotate the line A_5B_5 about the point C_5, the other quantities remaining unchanged, so that its inclination to AB is decreased. The point A_5 and the curve $A_5C_7B_6$ are thus moved upward and in consequence the work of the dynamic force is greater than the work of friction for the movement AB, which causes the stroke toward B to be lengthened. Therefore the arrangement in which the friction necessary to produce stability is just equal to the force AA_3, furnishes the limit for the inclination of the force line A_3B_3.

Now $CC_4 = AA_3 = \varepsilon M_0$ and the sum of the static forces and inertia forces from (7) and (8) is

$$AA_3 = a\,\theta_a = z\,\delta\,\frac{\theta_a}{\theta_r}\,M_0 + \frac{J'}{J_s}\,M_m\,\frac{\theta_a}{\theta_r}$$

Equating with the equation above

$$\varepsilon = z\,\frac{\theta_a}{\theta_r}\left[\delta + \frac{J'}{J_s}\cdot\frac{M_m}{z\,M_0}\right] = z\,\frac{\theta_a}{\theta_r}\left[\delta + \frac{I''}{I_0}\right]$$

But since ε must still fulfill the condition of stability (17) we have

$$z\,\frac{\theta_a}{\theta_r}\left[\delta + \frac{I''}{I_0}\right] = \frac{\pi}{2}\,\frac{\theta_a}{\theta_r}\cdot\frac{T}{I_0\sqrt{\delta + \frac{I''}{I_0}}}$$

from which $$\delta + \frac{I''}{I} = \sqrt[3]{\left(\frac{\pi}{4}\right)^2\frac{I^2}{I_0^2}} = .85\sqrt[3]{\frac{T^2}{I_0^2}}. \qquad (18)$$

Thus if δ or $\frac{I''}{I_0}$ be chosen smaller than is called for by this equation a stable governor is impossible. Equation (18) may be conveniently termed as the limiting condition for δ while the stability condition is expressed by (17a), which when $\frac{\theta_a}{\theta_r}$ is taken as $\frac{1}{4}$ or the arbitrary ratio of the load increments,

$$\varepsilon^2 \geq \left(\frac{\pi}{8}\right)^2\frac{T^2}{I_0(\delta I_0 + I'')} \qquad (17c)$$

C. Maximum Velocity Change.

While the governor must satisfy the condition of stability, it must also be arranged so that with a definite change of load the change of speed of the engine will remain within certain definite limits, usually 2%, with 30%

decrease of load. The determination of the maximum speed change will be obtained by the solution of the differential equations of motion.

Acting on the governor are the inertia forces viz: the centrifugal force with moment M_c and the tangential inertia force with moment M_g, the spring moment M_s, the brake moment M_B, the variable friction M_r, and a moment M_s due to the resistance of the valve gear. The latter consists of a constant part which may be embraced in the spring calculation, and a periodic part, which in modern shaft governors is sufficient to produce a periodic oscillation of the pendulum about its position of equilibrium. The effect of internal friction is merely to retard the motion of the pendulum, so we may neglect M_r and M_s if we refer the oscillations to the mean equilibrium position. Accordingly, let θ equal the angle traversed by the pendulum from its original position and ω_0 equal the original angular velocity. The equation of motion we obtain from the principle of work, from which the increase of kinetic energy is equal to the work put into the system. If ω_r denotes the relative angular velocity of the pendulum, the

kinetic energy is $\frac{1}{2} J \omega_r^2$ and the increase, $J\omega_r^2 \delta + \frac{1}{2}\omega_r^2 \delta J$, must be equal to the element of work $\Sigma M \omega_r dt$.
Assuming J to be invariable we have the usual form, $J\frac{d\omega}{dt} = J\frac{d^2\theta}{dt^2} = M_s + M_g + M_i + M_b$ (19)

For the motor or engine masses, the moment M_p of the driving force and M_g of the resistance come into consideration. The first is variable, there being a constant part independent of the governor position, and a periodic part, which supports the oscillations of the pendulum. The velocity will show periodic oscillations, which may be neglected with smaller error, the greater the speed of the motor. Under the same circumstances, we must neglect the unsteadiness due to the fact, that the governor must wait from ad mission to cut-off before a change in force is possible. We have then,

$$J_s \frac{d\omega}{dt} = M_p - M_g \qquad (20)$$

Let ω_g denote the velocity for equilibrium for the amplitude θ of the governor. The moment of the centrifugal force is $M_f = M_{eh}\omega^2$ using the notation of Fig. 3. The moment of the spring tension, as will be shown later, for the same position is equal to the centrifugal moment for the equilibrium velocity, that is,

$$M_i = - M_{eh}\omega_g^2$$

Denoting by δ, the coefficient of fluctuation, we have $\omega_g = \omega_0 \left(1 + \delta \frac{\theta}{\theta_r}\right)$

The quantity $\delta^2 \frac{\theta^2}{\theta_r^2}$ is small, both δ and $\frac{\theta}{\theta_r}$ being small proper fractions. Neglecting this quantity,

$$M_s + M_i = 2 M e h \omega_0^2 \left(\frac{\omega - \omega_0}{\omega_0} - \delta \frac{\theta}{\theta_r}\right) \qquad (21)$$

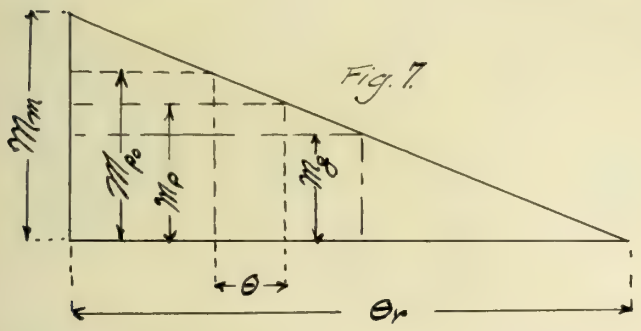


Fig. 7.

Initially $M_{p_0} = M_{g_0}$ but M_{g_0} suddenly changes to the smaller and constant value M_g as can be seen from Fig. 7, and we derive from the rectilinear force curve, with amplitude θ as seen in the figure,

$$-M_g + M_p = M_{g_0} - M_g - M_m \frac{\theta}{\theta_r}$$

These values inserted in (19) and (20) give

$$J \frac{d^2\theta}{dt^2} = 2 M e h \omega_0^2 \left[\frac{\omega - \omega_0}{\omega} - \delta \frac{\theta}{\theta_r}\right] + J \frac{d\omega}{dt} - K \frac{d\theta}{dt} \qquad (19a)$$

and

$$J_s \frac{d\omega}{dt} = (M_{g_0} - M_g) - M_m \frac{\theta}{\theta_r} \qquad (20a)$$

In order to give the equations a convenient form we introduce the following notation.

$\xi = \frac{\omega - \omega_0}{\omega_0}$ = relative change in speed in terms of the initial speed.

$\eta = \frac{\theta}{\theta_r}$ = relative motion of the governor in terms of the total motion.

$\Delta = \frac{M_{g_0} - M_g}{M_m}$ = load change in % of max. load.

Further let $M_0 = Mehw_0^2$ with a constant mean value for whole motion, i.e. $h = h_0$, the distance of the C. of G. from the initial position.

$M_g = J' \frac{dw}{dt}$ where J' is considered constant

Introducing these relations, with the coefficients T_0, T, T', and T'' previously used, equations (19a) and (20a) become

$$T^2 \frac{d^2\eta}{dt^2} = \zeta - \delta\eta + T'' \frac{d\zeta}{dt} - T' \frac{d\eta}{dt} \qquad (22)$$

$$T_0 \frac{d\zeta}{dt} = \Delta - \eta \qquad (23)$$

Each member of these equations represents an abstract number. The final values corresponding to a new inertia (so far as possible) we find by making ζ and η constant in (22) and (23)

$$\eta = \Delta, \quad \zeta = \delta\Delta \qquad (24)$$

Let $\zeta = \zeta_e + \zeta'$, $\eta = \eta_e + \eta'$ (25)

In place of t, introduce a new variable τ connected by the following relations

$$\tau = \frac{t}{T_2}, \qquad T_2 = \sqrt[3]{T_0 T^2} \qquad (25a)$$

By differentiation

$$T_0 \frac{d^2\zeta}{dt^2} = \frac{d\eta}{dt} \qquad dt = z\, d\tau$$

$$\frac{T_0}{T_2} \frac{d^2\zeta}{dt^2} = \frac{d\eta}{dt} \qquad \frac{T_0}{T_2} \frac{d^3\zeta}{dt^3} = \frac{d^2\eta}{dt^2}$$

then $$\frac{T^2 T_0}{T_2} \frac{d^3\zeta}{d\tau^3} = \zeta - \frac{\delta T_0}{T_2} \frac{d\zeta}{d\tau} + \frac{T''}{T_2} \frac{d\zeta}{d\tau} - \frac{T' T_0}{T_2^2} \frac{d^2\zeta}{d\tau^2}$$

Let $x = \frac{T' T_0}{T_2^2}$, $y = \left(\frac{\delta T_0}{T_2} + \frac{T'}{T_2}\right) = \left(\delta + \frac{T''}{T_0}\right) \frac{T_0}{T_2}$ (26)

Then, $\frac{d^3\bar{\beta}'}{d\tau^3} + X\frac{d^2\bar{\beta}'}{d\tau^2} + Y\frac{d\bar{\beta}'}{d\tau} + \bar{\beta}' = 0$ (27)

The character of the regulation depends upon X and Y. The solution of (27) necessitates the determination of the roots of the cubic equation

$$\varphi^3 + X\varphi^2 + Y\varphi + 1 = 0 \qquad (28)$$

which has one real root and two complex roots $\varphi_1 = \varphi_0$, $\varphi_2 = u + iv$, $\varphi_3 = u - iv$ and from these we obtain first $\bar{\beta}'$ and then $\bar{\beta}$,

$$\bar{\beta} = \bar{\beta}_e' + \bar{\beta}' = \delta\Delta + \lambda_0 e_0^{\varphi_0\tau} + [\mu \cos v\tau + \nu \sin v\tau]e^{u\tau} \qquad (29)$$

The constants λ_0, u, v are determined from the original conditions from which $t = 0$, $\tau_0 = 0$ $\omega = \omega_0$ and $\eta = 0$. The derivative of $\bar{\beta}$ for (29) is obtained as an indirect function thus:

$$\left[\frac{d\bar{\beta}}{dt}\right]_{t=0} = \left[\frac{d\bar{\beta}}{d\tau}\cdot\frac{d\tau}{dt}\right]_{t=0} = \frac{1}{T_E}\left[\frac{d\bar{\beta}}{d\tau}\right]_{\tau=0} = \frac{\Delta}{\tau_0}$$

Finally for $t = 0$, $\frac{d\eta}{dt} = 0$ from which

$$\frac{d^2\bar{\beta}}{dt^2} = \left[\frac{d^2\bar{\beta}}{d\tau^2}\right]_{t=0} = 0$$

We have therefore the following equations

$$\left.\begin{aligned} &\lambda_0 + \mu = -\delta\Delta \\ &\lambda_0 + u\mu + v\nu = \frac{T_E}{\tau_0}\Delta \\ &\lambda_0\varphi_0^2 + (u^2 - v^2)\mu + 2uv\nu = 0 \end{aligned}\right\} \qquad (30)$$

The governor is serviceable in case φ_0 and u are negative, since in that event the

velocity attains the limiting value $\bar{\zeta} = \delta\Delta$ with diminishing oscillations. This is true if ψ_0 and u are negative because in 29, $\lambda e^{\rho_1 T}$ and $[\quad] e^{uT}$ will be proper fractions and as ψ_0 and u increase, the fractions become smaller with zero as the limit. This occurs when

$$X \cdot Y > 1 \qquad (31)$$

If $X \cdot Y \leq 0$, the oscillations increase from $\bar{\zeta}$ and equilibrium is impossible. Relation (31) represents therefore the stability condition of the motor action and when written with the coefficients of T becomes

$$T'(\delta T_0 + T'') > T^2 \qquad (32)$$

which agrees with what has been previously obtained.

In order to obtain the maximum velocity change, we differentiate equation (29), solve for the value of T, and substitute back the value obtained. The equation shows that $\bar{\zeta}$ is proportional to Δ for in (30) λ_0, u, v, are directly proportional to Δ. If we assume $\delta = 0$, $\frac{T\bar{\zeta}}{T_0}$ appears as a common factor since $\bar{\zeta}$ is proportional to $\Delta \frac{T\bar{\zeta}}{T_0}$. Now, when $\delta = 0$, ψ_0, u, v, depend upon X and Y and $\bar{\zeta}$ has the form

$$\bar{\zeta} = \Delta \frac{T\bar{\zeta}}{T_0} \qquad (33)$$

If now we replace $\bar{\zeta}$ by the variable

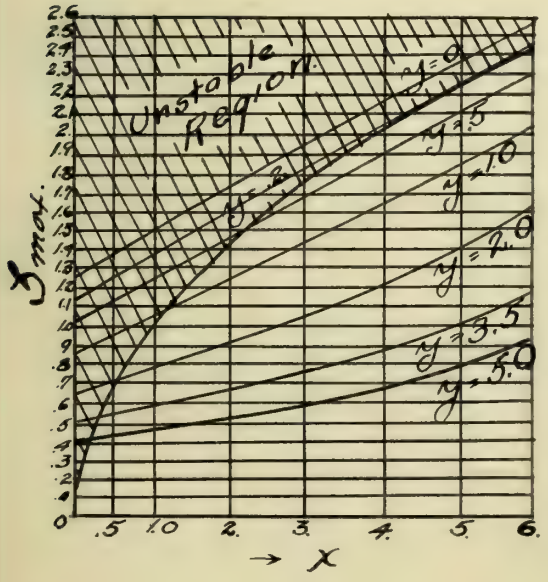
$$\bar{S} = \frac{\bar{S}}{\Delta} \cdot \frac{T_0}{T_2}, \qquad (34)$$

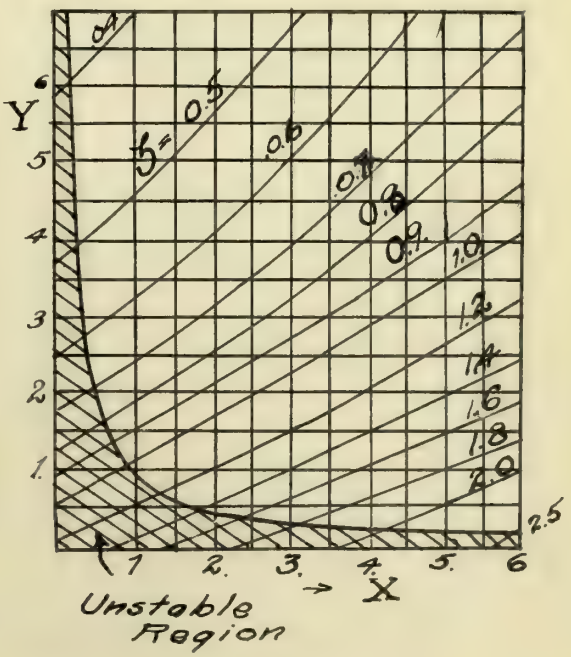
we find that it depends upon X, Y, and T only. We find therefore that

$$S_{max} = \frac{T_e}{T_2} \cdot \frac{\bar{S}}{\Delta}_{max} = \text{function } X, Y, \qquad (35)$$

The problem is then reduced to one of two variables. The values of (S_{max}) have been compiled by Stodala and are laid off by the following charts. Intermediate values are obtained by interpolation. The curves Y = constant in the first chart are curved so little, that they may be taken as straight lines without serious error. We thus obtain the empirical formula

$$S_{max} = \frac{1.95 + 0.18Y}{1.66 + Y} + (.21 + .027Y)X \qquad (36)$$





In applying this theory to machines of given power, the maximum coefficient of fluctuation of speed during a revolution is

$$\delta_{max} = \frac{\omega_{max} - \omega_{min}}{\omega_0},$$

in which δ has nothing to do with the fluctuations of the governor. Let A be the work of the greatest effort per revolution of the engine, and ΔA, the maximum excess of the work of the steam over that of the constant resistance; then we obtain

$$\psi = \frac{\Delta A}{A} \qquad (37)$$

which may be called the coefficient of fluctuation of the driving force. Since through ΔA there results a change in kinetic energy, we have to express ΔA in finite differences from the differential formula $Jw\,dw = dE$, thus

$$\Delta A = J\omega_0(\omega_{max} - \omega_{min})$$

If n_1 = revolutions per second, we have,

$$A = M_{pm}\omega_0 \text{ and dividing}$$

$$\psi = \frac{\Delta A}{A} = \frac{J\omega_0}{M_{pm}}\left[\frac{\omega_{max} - \omega_{min}}{\omega_0}\right] n_1$$

The first factor is the time T_0, and the second δ_m; solving for T_0

$$T_0 = \frac{\psi}{\delta_m n_1} \qquad (38).$$

ψ in ordinary cases with single engines

varies from .3 to .18. As stated previously, $T'' = \frac{J'\omega_0}{2M_0}$, where J' is the expression given on page 10 and where $M_0 = Me h_0 \omega^2 \theta_0$.

Then, $T'' = \frac{1}{2eh_0\omega_0}\left(\frac{v_2}{M} + r_0\theta_0\right)$. (39)

$T^2 = \frac{J\theta r}{2M_0}$ as stated previously which reduces to,

$$T^2 = \frac{J}{2M} \cdot \frac{\theta r}{eh_0\omega_0^2} \qquad (40)$$

With these equations and the charts, the problem of max. velocity oscillation can be easily solved.

d. Static Conditions.

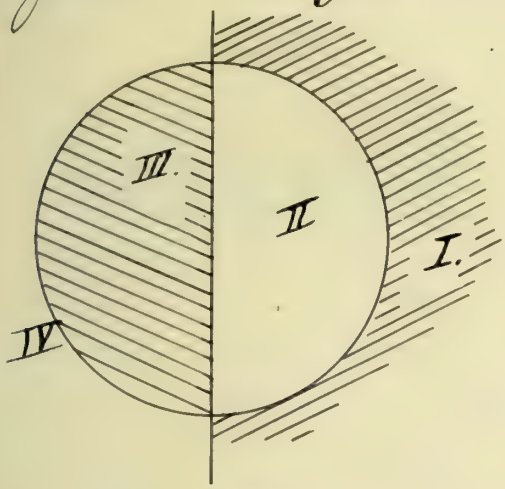
The question of stability and maximum velocity oscillations having been discussed, it is necessary to turn back to static considerations. Besides the moments of the acceleration forces is the moment of the centrifugal force, $Me h\omega^2$. The force $Mr\omega^2$ is always in existance when the engine is running and its moment about O, the pivot of the governor, must be balanced by the moment of some external force which usually takes the form of a spring. The moments $J\alpha$ and $Mrc\alpha$ however spring into existance only when there is a change in velocity of the fly wheel.

Suppose that the wheel and attached mass, Fig. 3, is rotating clockwise with an angular speed ω, which for the moment is constant. The forces acting on the governor mass are; (1) the centrifugal force $M r \omega^2$, (2). the spring pull, (3) the reaction at the pivot O. The mass is held in equilibrium and therefore the moment M_s of the spring tension balances the moment $M_c = M e h \omega^2$ of the centrifugal force.

Suppose now the fly-wheel increases its speed to ω_1, the mass remaining in the same position relative to the wheel. The centrifugal force now is $M r \omega_1^2$ and the moment about O is $M e h \omega_1^2$. This exceeds the moment $M_c = M_s = M e h \omega^2$ and in consequence, there is an unbalanced moment whose magnitude is $M e h (\omega_1^2 - \omega^2)$. But the increase from ω to ω_1 implies a certain angular acceleration α, so that in addition to the moment just given, we have also the moments $J\alpha$ and $M r c \alpha$. These three moments if they have the same sense, unite in moving the governor mass to a new position. Whether they have the same sense depends upon the location of G relative to O and S, Fig. 3. In Fig. 8, let a circle be drawn upon OS as a

diameter thus dividing the space in which G may be located into four regions as shown, then the following table gives the signs of the moments when G is in each of the four regions.

Fig. 8.



Region	1	2	3	4
$Mrc\alpha$	+	−	−	+
$I\alpha$	+	+	+	+
$Meh(\omega_1^2-\omega^2)$	+	+	−	−

In the inertia governor, the centrifugal force is desired just large enough to keep the engine at the desired speed, and the regulation devolves upon the inertia action. The decided advantage of the inertia type over the purely centrifugal type lies in the almost instantaneous action of the forces as could be seen during the discussion regarding stability.

As stated previously, the centrifugal force must be balanced by a spring tension and the moments of the two forces about the pivot O must be equal. Thus

When $w^2 = T.l$, where T represents the spring tension and l, the arm of T, that is the perpendicular distance from O to the line of action of the spring. The simple arrangement of Fig. 9. shows how the equilibrium is obtainable.

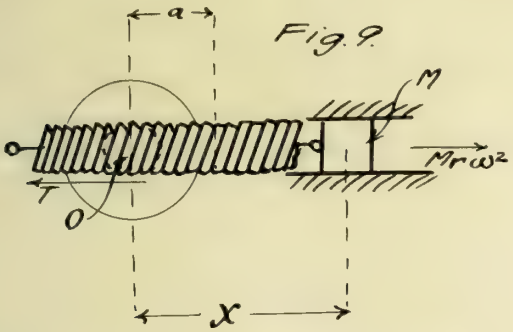


Fig. 9.

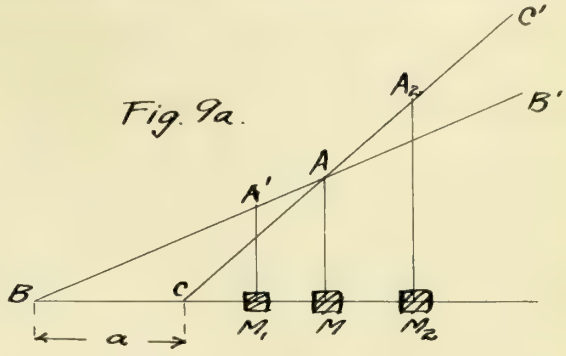


Fig. 9a.

The centrifugal force of the mass M is $M \times w^2 = F$
The spring tension is $S(X-a) = T$, where a is the distance of M from the center, where the spring is unstrained, and S is the spring strength. These two equations, are equations of straight lines with their origin at B, Fig. 9a, and are represented by CC' and BB' respectively. Thus with the mass at M, the two forces are equal, at M_1 the centrifugal force is in excess and at M_2 the spring tension is in excess. In both cases the unbalanced force tends to bring the mass M to the position of equilibrium.

The forces of the governor being now defined, the next step to be considered, is the location of the important points O, S, and

G and the point of application of the spring tension. The valve and valve seat of the engine were measured as is shown in a subsequent plate and a Zeuner diagram was drawn which would give the best possible results with regard to steam distribution. Then the point O, the pivot, and the path of the eccentric center were definitely located. The diagram was drawn to give a maximum cut-off at five-eighths stroke, and for for this position of maximum cut-off, the governor arm must swing through an angle θ.

By means of Fig. 8 and the corresponding table, the point G, the center of gravity of the governor was located and from the Zeuner diagram the angle θ can be determined. Referring to Fig. 10, the location of the points O, S, G and M is readily seen. The lengths p and p, are the moment arms of the spring tensions OM and OM', M and M, being the point of application of the

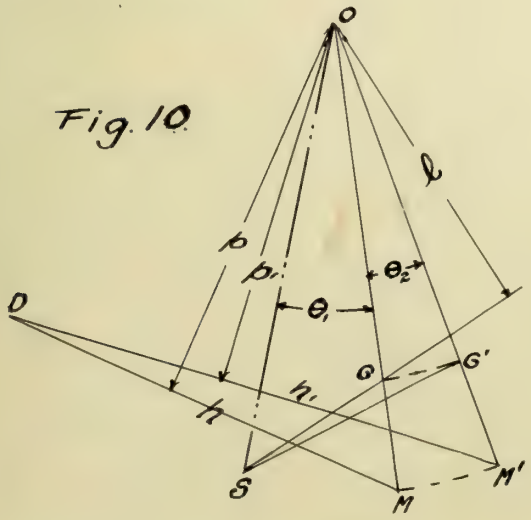


Fig. 10.

spring for extreme positions. The angle θ_2 is that through which the governor swings from no load to full load position. The angle θ_1 is the angle which the line OG makes with the line OS for its initial position. The spring length changes from $OM = h$ to $OM' = h_1$. Let h_0 represent the length of the unstrained spring. Then $T_M = S(h - h_0)$ and $T_{M'} = S(h_1 - h_0)$ where S = strength of the spring.
The moment of T_M about $O = Sp(h - h_0)$.
The moment of $T_{M'}$ about $O = Sp'(h_1 - h_0)$.
As the centrifugal moments depend upon the area SGO, and are proportional to it, and as these are balanced by the moments of the spring tension, it follows that,

$$Sp'(h_1 - h_0) = OG' \sin(\theta_1 + \theta_2)$$

$$Sp(h - h_0) = OG \sin \theta_1 \qquad OG = OG'$$

Dividing, $\frac{p'}{p} \frac{h_1 - h_0}{h - h_0} = \frac{\sin \theta_1 + \theta_2}{\sin \theta_1}$

As soon as a point D is chosen, h_0 can readily be determined. The spring must be chosen to satisfy the equation above and also to satisfy certain physical conditions which enter into the case. For instance the spring should not be extended more

than 30% of its length when unstrained. To obtain the best results and to keep the tension in the spring as small as possible, the angle OMD is made 90° or nearly so. The angle θ_1 is then chosen by cut-and-try methods, and the best results are obtained when the angle $(\theta_1 + \theta_2)$ does not exceed 40°. This in a measure determines θ_1. In the present case θ_1 is made 20° and θ_2 is 18°. The size and the strength of the spring then depends upon the value of S. In the present case $h_1 = 12.2"$ and $h = 11.0"$, and h_0 was found to be 9.4" as the following relation shows.

$$\frac{p_1}{p} \cdot \frac{h_1 - h_0}{h - h_0} = \frac{\sin 38°}{\sin 20°}$$

p was taken equal to p_1 as the angle GOS was taken 90° for mid-position.

Then,
$$\frac{12.2 - h_0}{11.0 - h_0} = \frac{\sin 38°}{\sin 20°}$$

whence,
$$h_0 = 9.4".$$

Different points of application of the spring were taken. As the point D, was moved away from the pivot O, h_0 became larger, the percent elongation became smaller, but the size of the enclosing

fly-wheel made this impossible. The point was then moved toward O, but the initial tension was found to be too high, involving too heavy a spring. The angle θ, was varied from 10° to 70°, but below 20° it necessitated moving the center of gravity away from the pivot, which caused other difficulties in regard to the inertia action. As θ increased, the spring tensions became too large, and too long a spring would be required.

III. Application of Theory to the Design.

The energy stored up in the governor necessary to drive the valve mechanism (denoted by E) is equal to the work required to stretch the spring. If T_1 is the tension in the spring at M, Fig. 8., and T_2 that at M', then $E = \frac{T_1 + T_2}{2}(h_2 - h_1)$

The values generally taken for E vary from 0.5 to 8.0 foot-pounds per horse power. The maximum horse power developed by the engine in question is 6 H.P. Therefore the energy to be stored up is 6 × 5.0 foot pounds = 30 foot pounds, 5.0 foot pounds being

chosen as the amount to be stored up per horse power.

$$h_2 - h_1 = 1.2 \text{ in.} = 0.1 \text{ ft.}$$

Then $\frac{T_1 + T_2}{2} = \frac{30}{0.1} = 300$ = mean tension in spring.
The elongation at mid-position is

$$\frac{12.2" + 11.0" - 9.4"}{2} = 2.2"$$

$300^{\#} \div 2.2 = 136^{\#}$ = strength of spring.

As was stated previously, the centrifugal force is balanced by the spring tension, the moments of the two forces about O being equal. The moment of the centrifugal force, Fig. 8, at G is $M \cdot \overline{5G} \cdot \omega^2 \cdot l = M e_h \omega^2$ in Fig. 3, and the moment of the spring tension, Fig. 10, is $(h - h_0) Sp$. Thus we have, $M \omega^2 l \cdot \overline{5G} = (h - h_0) Sp$, of which all except M are known. It is thus an easy matter to determine M. Take the case of the governor in the position G.

$$S = 136$$

$$h - h_0 = 11." - 9.4" = 1.6", \quad p = 3.8"$$

$$e_h = \overline{5G} \cdot l = 3.4 \times \frac{1.2}{12}$$

$\omega^2 = 686$ since the number of revolutions is 250 per minute.

Then, $136 \times 1.6 \times 3.8 = 3.4 \times \frac{1.2}{12} \times 686\, M$

or, $M = 3.6$

For mid-position,

$$136 \times 2.2 \times 3.8 = 3.4 \times \frac{1.7}{12} \times 686\ M$$

$$M = 3.5.$$

For the extreme position, G',

$$136 \times 2.8 \times 3.8 = 3.4 \times \frac{2.2}{12} \times 686\ M$$

$$M = 3.4.$$

The mean variation of M is 3% and the weight of the governor since $M = 3.5 = \frac{W}{32.2} = 114$ lb. In other words the governor must weigh 114 pounds, in order that in moving through its short arc, 30 foot-pounds may be stored up.

The maximum tension in the spring is $136^{\#} \times 2.8 = 380^{\#}$ for its extreme position. A spring must be chosen which will stand a direct tension of 380 lb and will also have the desired strength, 136 lb per inch elongation.

The length of the spring before elongation = 9.4"
" " " " " " after " " = 12.2"

Extension = 2.8" = 29%

Using $\frac{5}{16}$" wire, in 9.4" there are 30 coils.
The elongation per coil is 2.8" ÷ 30 = .093"
" " " " " per 100 lb = .093" ÷ 3.8 = .025"
With $\frac{3}{8}$" wire, the elongation per coil per 100 lb is .029". From the table on springs (Kent, pages 349-350), a steel spring $\frac{5}{16}$" wire, $1\frac{3}{4}$" in diameter or a $\frac{3}{8}$" wire $2\frac{1}{2}$" in diameter

is required, the maximum tension to which either may be subjected being 500# either satisfies the conditions but in order to secure compactness the $\frac{5}{16}$" spring $1\frac{3}{4}$" in diameter was chosen.

With all the points known upon which the design of the governor depends, the next step is to determine upon the proportions of the mass itself. The fly-wheel within which the governor is to be enclosed is 26" inside diameter, so that in order to allow it to swing through its arc, its maximum length can not be over 24". It is then necessary to get 114# within 24" in length, and still obtain as large a moment of inertia as possible. Different forms were chosen with different widths and thicknesses, and one was finally chosen with the proportions shown in the following plates. The polar moment of inertia was determined by ordinary methods. The polar I of the entire mass about the center of gravity is

$$77 \times 3.5 = 269 = Mr^2$$

$$r^2 = 77, \qquad r = 8.77"$$

The moment about the pivot = 317, r being 9.5"

After determining J and r, it is necessary to test the governor for stability and find the maximum velocity oscillations, by the principles of Stodola using the values just found. ψ will be assumed as 0.3, being the excess of the force over resistance, and $\delta_{max} = 0.1$ being maximum coefficient of fluctuation of the governor.

$$\psi = 0.3, \quad \delta_{max} = .01 \qquad N = \frac{250}{60} \qquad \omega = 26.2.$$

Then, $$T_0 = \frac{.3}{.01 \times \frac{25}{6}} = 7.2 \qquad \text{from (38).}$$

and $$T_2 = \frac{77 + 1.1}{2 \times 3.6 \times 1.8 \times 26.2} = .231 \qquad \text{from (39).}$$

" $$T^2 = \frac{1}{2} \cdot \frac{317}{3.5} \cdot \frac{31}{3.6 \times 1.8 \times 686} = .00316 \qquad \text{from (40).}$$

" $$T_2 = \sqrt[3]{7.2 \times .00316} = .283 \qquad \text{from (26a)}$$

" $$Y = \frac{.231}{.283} = .82 \text{ approx.} \qquad \text{from (27).}$$

By choosing $x = 1.5$, the table gives $\beta = 1.2$, when $Y = .82$ as found above. This shows that the governor is stable.

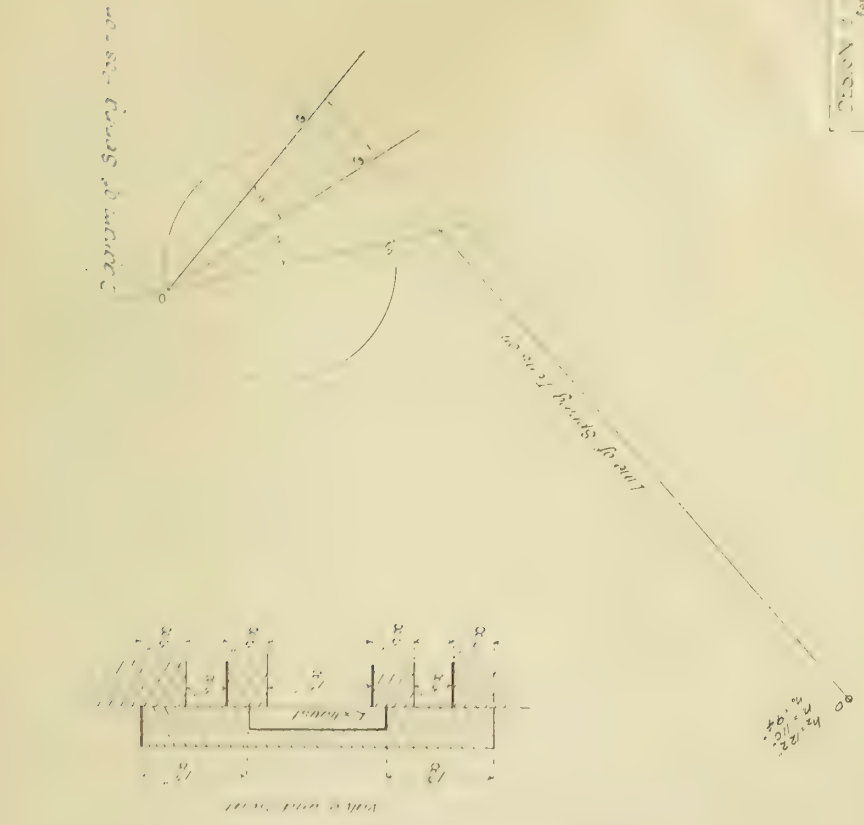
Then by (36) the maximum change in speed for a change of load of 50% is,

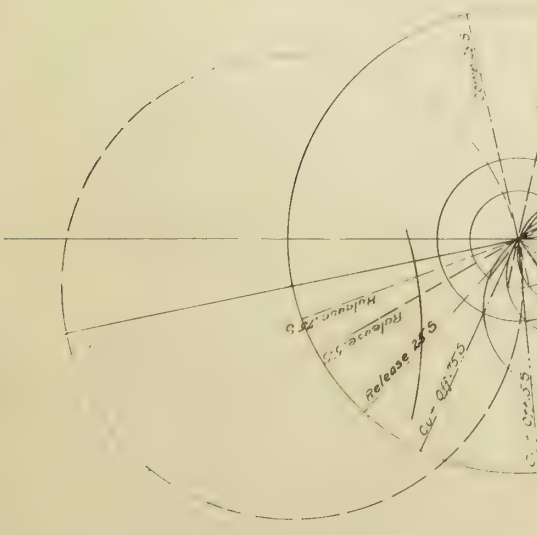
$$\frac{.281}{7.2} \times 1.2 \times 50 = .019$$

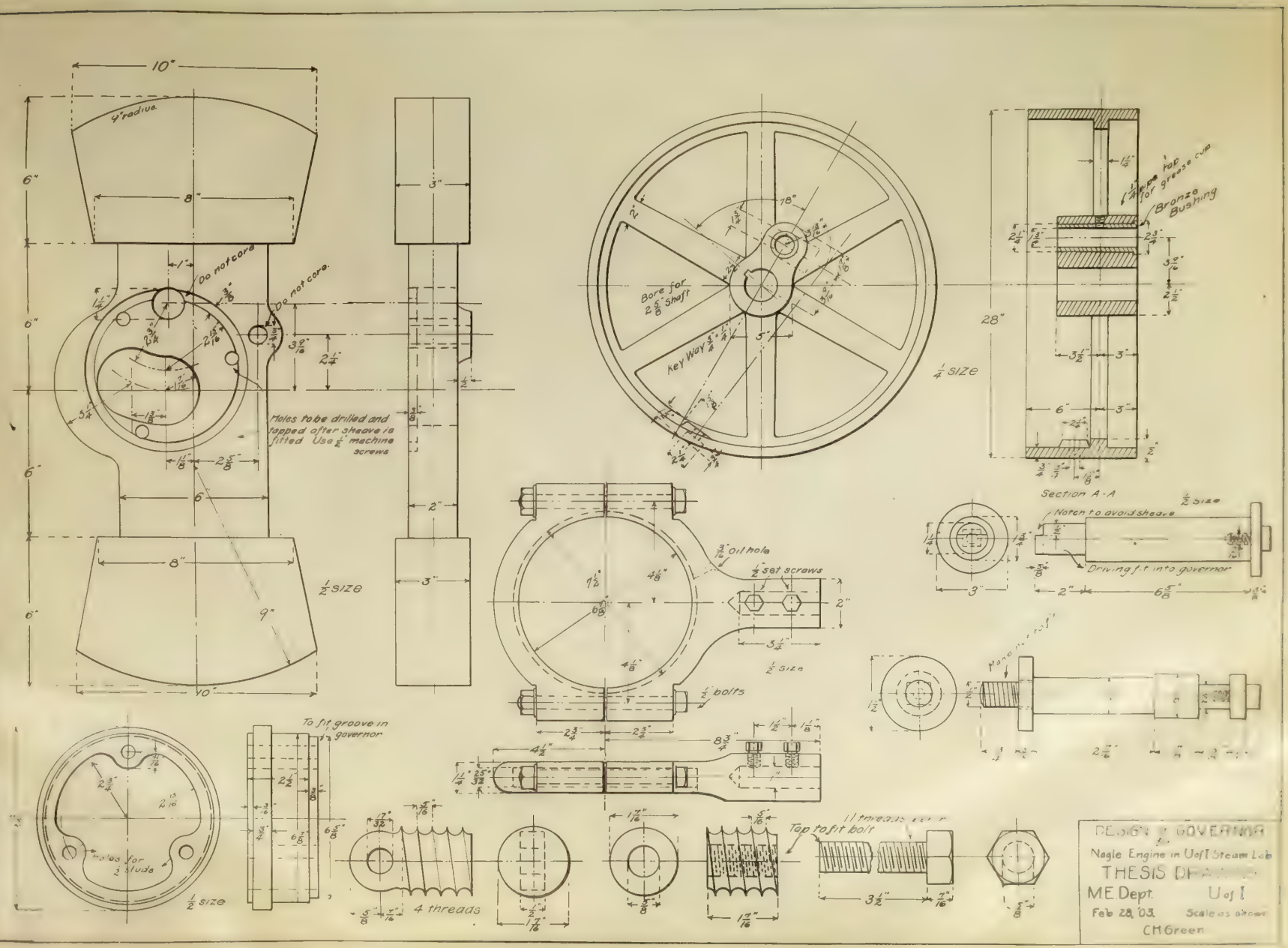
showing that with a change of load of 50% the engine speed will vary through about 2%.

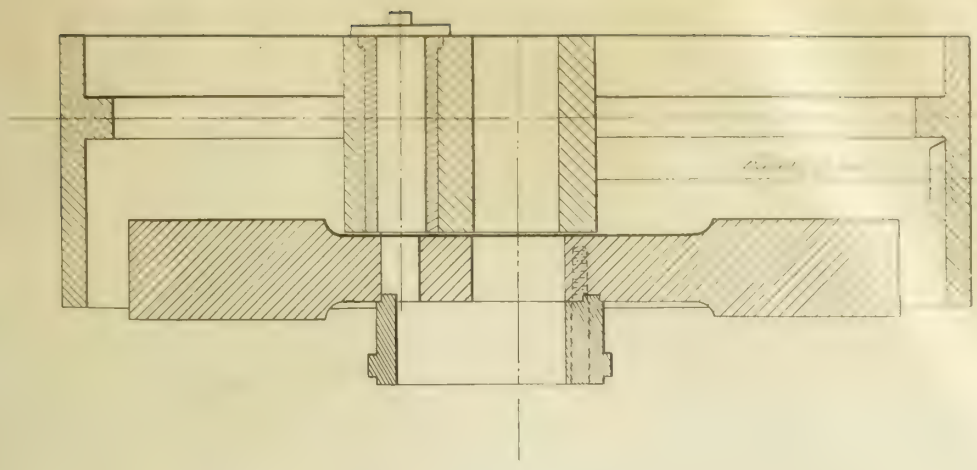
With the Rites type of governor, having the valve rod attached directly to the governor mass, it is necessary that the resisting moment of the valve mechanism shall be small in comparison with the moment of the governor forces. In the case of the plain slide valve under ordinary circumstances, the resistance is such that its moment brings about undesirable conditions which prevent the governor from being sensitive. In order to avoid this, a balanced valve, with very slight resistance was designed as is shown in a subsequent plate. Previous to the design of this balanced valve however, it was decided to place guides in the fly wheel, which would take up the direct tension of the valve rod, but the friction along these guides would destroy the sensitiveness of the governing action, which is its essential point of vantage.

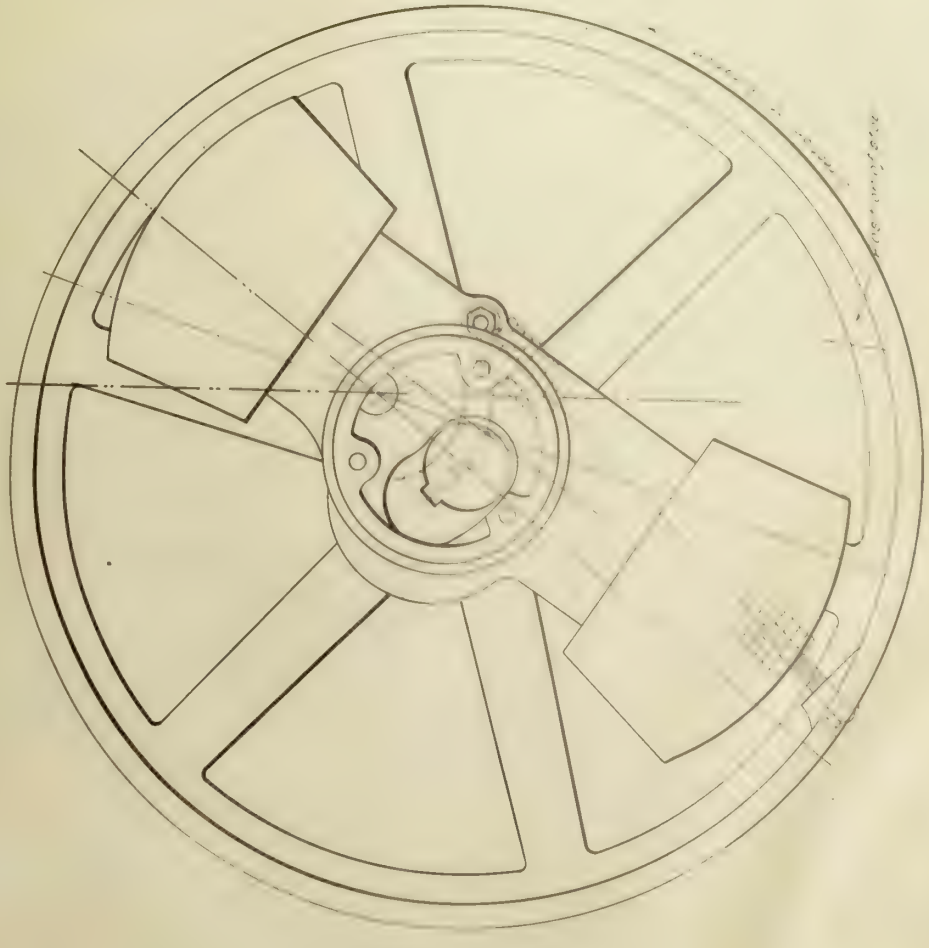
DESIGN OF GOVERNOR
Nagle Engine Unit Steam Lab
THESIS DRAWING
ME Dept Unit
CH Green

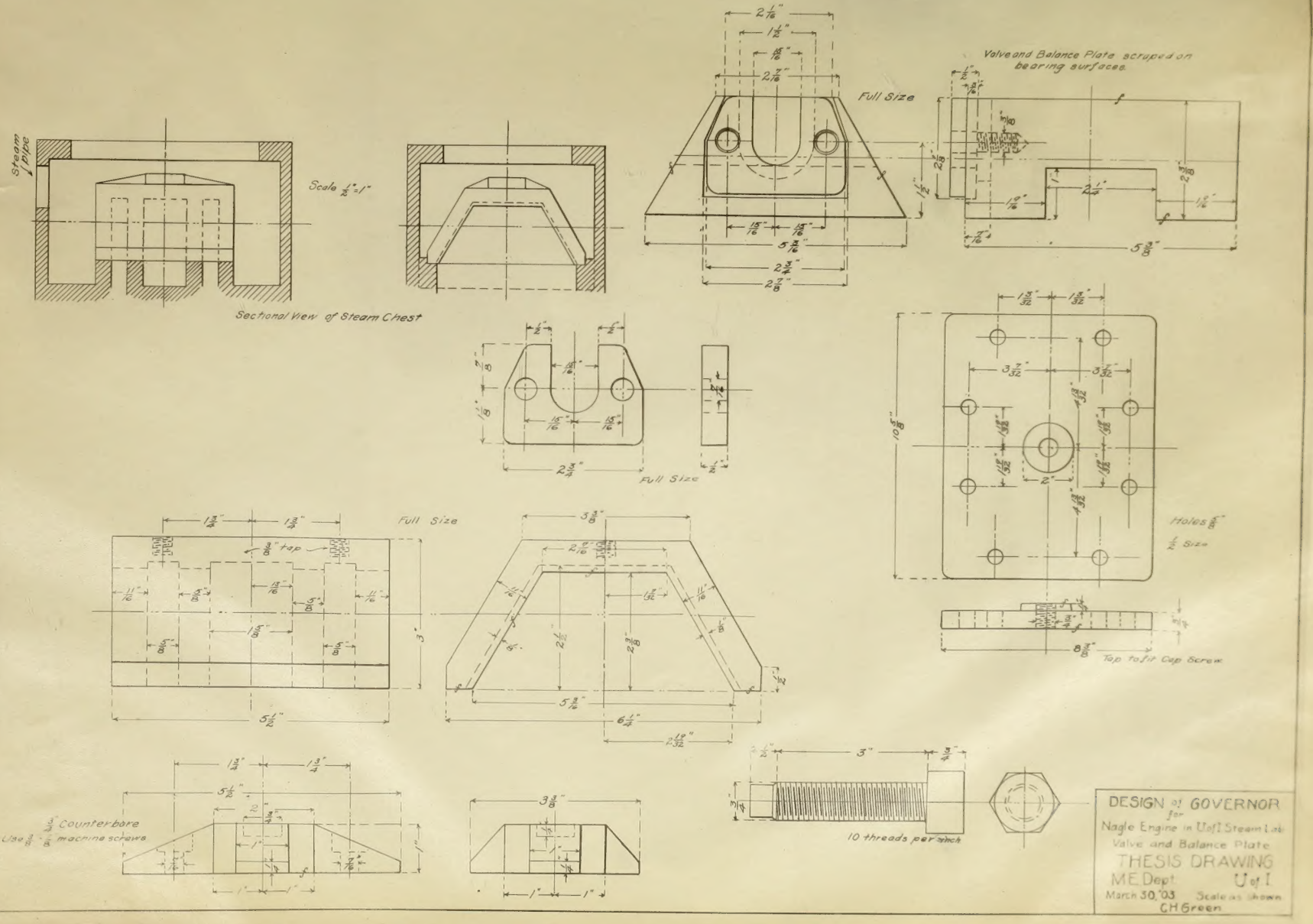





10"
9" radius
6"
8"
3"
Do not core
Do not core
Holes to be drilled and tapped after sheave is fitted Use ½" machine screws
½ SIZE
9"
Bore for 2⅝" Shaft
Key Way ¾" x ¼"
18°
28"
¼ SIZE
Bronze Bushing
Section A-A
½ Size
Notch to avoid sheave
Driving f.t into governor
3" oil hole
½" set screws
½ bolts
½ Size
To fit groove in governor
Holes for ¾ studs
½ size
4 threads
11 threads per "
Top to fit bolt
DESIGN OF GOVERNOR
Nagle Engine in U of I Steam Lab
THESIS DRAWING
ME. Dept. U of I
Feb 28, 03. Scale as shown
C.H. Green






Steam Pipe
Scale 1/2" = 1"
Sectional View of Steam Chest
Full Size
Valve and Balance Plate scraped on bearing surfaces.
Full Size
Full Size
Holes 5/8" ⌀
1/2" Size
Tap to fit Cap Screw
3/4" Counterbore
Use 3/4" - 5/8" machine screws
10 threads per inch
DESIGN of GOVERNOR
for
Nagle Engine in U of I Steam Lab
Valve and Balance Plate
THESIS DRAWING
M.E. Dept U of I
March 30, '03 Scale as shown
C.H. Green

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